# Measuring the Efficiency of Academic Units at the Teacher Training University

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**Abstract**

This study analyzes the performance of academic units at the Teacher Training University (TTU), Tehran, IRAN. The Data Envelopment Analysis (DEA) method is used to obtain the efficiency ratings for the University's faculties and departments. The technique based on linear programming and makes it possible to compare the efficiencies of Decision Making Units (DMUs) having multiple inputs and outputs without requiring prices. Efficiency ratings are calculated for 19 academic units, using two inputs and two outputs.

**(DEA, DMUs, Efficiency, Linear Programming)**

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## 1. Introduction.

The Teacher Training University has 21 academic units. Teaching and research are the main responsibilities of these units. In some cases these units are departments, such as Mathematics and Chemistry., These departments are part of the Faculty of Sciences. Other units are institutes that are not part of a faculty, for example, the Institute of Mathematics.

This study deals with the determination of technical efficiency, based on the production of multiple outputs and multiple inputs. For a single output and a single input, efficiency may be expressed as input per output, and academic units may be compared. For multiple inputs and outputs, such a measure is no longer straightforward.

A relatively recent technique, called Data Envelopment Analysis (DEA) and developed by Charnes, Cooper, and Rhodes 1978 (CCR) [3] and Banker, Charnes, and Cooper 1984 (BCC) [2] extends the concept of technical efficiency from the case of one input and one output to that of multiple inputs and outputs. This approach is based on linear programming and allows the comparison of units producing the same outputs using the same inputs, declaring one or more to be efficient, and others inefficient. This technique is applied in this paper to inputs and outputs of 19 academic units at the Teacher Training Uni-

ing is more important at the Teacher Trai ing University. Products of teaching are e rollments of various courses at different level The enrollments of courses at B.Sc, M.Sc, ar Ph.D levels are known for every unit and a available at the computer center.

Products of research are scholarly pu lications in various forms. The number books and conference proceedings, and th number of refereed papers and technical r search reports are available. These can b considered as the products, or outputs of th unit. The inputs of an academic unit co sist of the academic staff, the support sta space, and other expenses. The crucial inpu is the academic staff, namely professors an instructors at various ranks.

Data are available for these inputs an outputs. In fact, there are too much dat for comparison. One possibility is to con centrate on one comparison only, for exam ple, on the number of papers per academi staff member. However, those units produc ing a significant number of books would no be happy. Another solution would be to com pare weighted inputs and outputs of the aca demic units. The problem is then what th weights should be.

The DEA technique was proposed to com pare the efficiency of units with multiple in puts and outputs for which no costs or price are available without requiring prassigned w

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## 2. The Determination of Efficiency by DEA.

Data Envelopment Analysis provides a measure of the efficiency of a decision making unit (DMU) relative to other such units, producing the same outputs with the same inputs. The units to be compared may be banks [1], hospitals [4], and in this study they are academic units.

The efficiency of a unit involves a comparison of its outputs with its inputs. If there is only one output and one input, the efficiency is expressed as the ratio between its input and its output. For example, the efficiency of a computer can be found by dividing the number of calculations by the amount of time used, which results in the efficiency measure of calculations per second.

If there are multiple outputs and multiple inputs, and prices and costs are available for each of these, both outputs and inputs can be expressed in terms of money, so they may be compared by taking the ratio of the value of outputs over the value of inputs. Comparison in terms of money leads to what is called economic efficiency.

If prices and costs are not available, it is more difficult to determine efficiency. In some cases it is possible to make statements regarding efficiency. If two units have the same quantities of the inputs and the second unit produces at least the same quantities of the the same quantities of outputs but one unit has the same or lower quantities of inputs and for at least one input a lower quantity, then the other unit is inefficient.

Instead of comparing one unit with each of the other units separately, we may compare its outputs and inputs with linear combinations of the units.

For example, in table 1, we may compare the inputs and outputs of unit 4 with linear combinations of the units. If the linear combination produces at least the same output as unit 4 does, but uses the same or less of each of the inputs, with less for at least one input, then that unit is inefficient. The last column in Table 1 represents a combination of 0.1990 times unit 1 plus 0.1047 times unit 2 plus 0.5236 times unit 3. This combination yields precisely the same outputs as those of unit 4, but has lower input quantities. Hence unit 4 is inefficient when compared with the other three units.

It is also possible to determine the efficiency of unit 4. Let us find weights of all units (including unit 4) such that the weighted combination produces at least the same quantities of outputs, then choose the maximum proportion of the weighted inputs to the inputs of unit 4. When this proportion is minimized with respect to the different sets of weights, then it defines the efficiency measure for unit 4. The maximum value for that is 1, because unit 4 is itself included in the com-

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| | unit1 | unit2 | unit3 | unit4 | combination |
|---|---|---|---|---|---|
| input1 | 12 | 20 | 17 | 15 | 13.382 |
| input2 | 8 | 10 | 12 | 10 | 8.921 |
| output1 | 300 | 500 | 550 | 400 | 400 |
| output2 | 35 | 45 | 35 | 30 | 30 |

Table 1. Four units with inputs and outputs illustrated.

portion takes place by linear programming. LINDO can be very useful for this purpose. For example, in Table 1 the minimum proportion is obtained when the corresponding weights are 0.1990 for unit 1 and 0.1047 for unit 2 and 0.5236 for unit 3, and so the efficiency measure for unit 4 is $Max(13.382/15, 8.921/10) = 0.8921$. For units 1, 2, and 3 the efficiency may be determined in the same way. It turns out that units 1, 2, and 3 are efficient, because their outputs cannot be produced with lower inputs of other units.

There exists an alternative approach to determine the efficiency measure, which seems entirely different but is equivalent to it. This approach and the reason for this equivalence is more completely explained in the next section. It may be called the value approach, because it assigns values to all inputs and out-

## 3. DEA Models and Definitions.

There are multiple outputs and multiple inputs, and there are no prices and costs. Denote inputs by $X_{ij}$, $i = 1, \ldots, m$ and outputs by $Y_{rj}$, $r = 1, \ldots, s$ for decision making unit $j$, $j = 1, \ldots, n$. As it was mentioned in section 2, the following method will be used for measuring the efficiency of unit 0, where $0 \in \{1, \ldots, n\}$.

The weights of units (including unit 0 ) from the different sets of weights should be found such that the weighted combination produces at least the same quantities of outputs of unit 0, then choose the maximum proportion of the weighted inputs to the inputs of unit 0. When this proportion minimized with respect to different sets of weights, then it defines the efficiency measure for unit 0.

Also , the following method can be used. The weights of units (including unit 0) should

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of the weighted outputs to the outputs of unit 0. When this proportion maximized with respect to the different sets of weights, then it defines the above efficiency measure for unit 0.

The above models can be formulated as follows:

$$\min \max_{i=1,\dots,m} \frac{\sum_{j=1}^{n} X_{ij}\lambda_j}{X_{i0}} \quad (1)$$

*subject to:*

$$\sum_{j=1}^{n} Y_{rj}\lambda_j \geq Y_{r0} \quad r = 1,\dots,s$$

$$\lambda_j \geq 0 \quad j = 1,\dots,n.$$

and

$$\max \min_{r=1,\dots,s} \frac{\sum_{j=1}^{n} Y_{rj}\lambda_j}{Y_{r0}} \quad (1')$$

*subject to:*

$$\sum_{j=1}^{n} X_{ij}\lambda_j \leq X_{i0} \quad i = 1,\dots,m$$

$$\lambda_j \geq 0 \quad j = 1,\dots,n.$$

(1) and (1′) are equivalent definitions of efficiency of unit 0, because their transformed linear forms are equivalent problems. So only definition (1) is used. (1) is a nonlinear programming problem and it can be transformed to the following linear programming problem:

$$z_0 \geq \frac{\sum_{j=1}^{n} X_{ij}\lambda_j}{X_{i0}} \quad i = 1,\dots,m$$

$$\sum_{j=1}^{n} Y_{rj}\lambda_j \geq Y_{r0} \quad r = 1,\dots,s$$

$$\lambda_j \geq 0 \quad j = 1,\dots,n.$$

or

$$\min \quad z_0 \quad (3)$$

*subject to:*

$$X_{i0}z_0 - \sum_{j=1}^{n} X_{ij}\lambda_j \geq 0 \quad i = 1,\dots,m$$

$$\sum_{j=1}^{n} Y_{rj}\lambda_j \geq Y_{r0} \quad r = 1,\dots,s$$

$$\lambda_j \geq 0 \quad j = 1,\dots,n.$$

The dual of problem (3) will be as follows:

$$\max \quad g_0 = \sum_{r=1}^{s} \mu_r Y_{r0} \quad (4)$$

*subject to:*

$$\sum_{r=1}^{s} \mu_r Y_{rj} - \sum_{i=1}^{m} \omega_i X_{ij} \leq 0 \quad j = 1,\dots,n$$

$$\sum_{i=1}^{m} \omega_i X_{i0} = 1$$

$$\mu_r, \omega_i \geq 0 \quad r = 1,\dots,s;\ i = 1,\dots,m.$$

It can be shown that (4) is equivalent to a linear fractional programming problem. In fact, the following transformations

$$\omega_i = t\,v_i \quad i = 1,\dots,m$$

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$$t^{-1} = \sum_{i=1}^{m} v_i X_{ij}$$

with $t > 0$ , gives:

$$\max \quad f_0 = \frac{\sum_{r=1}^{s} u_r Y_{r0}}{\sum_{i=1}^{m} v_i X_{i0}} \qquad (5)$$

$$\begin{aligned} &\textit{subject to}: \\ &\sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \leq 0 \quad j = 1, \ldots, n \\ &u_r, v_i \geq 0 \quad r = 1, \ldots, s; \; i = 1, \ldots, m. \end{aligned}$$

(5) represents the value approach which was mentioned in section 2. (5) is equivalent to the following form which is called the CCR ratio [3]:

$$\max \quad h_0 = \frac{\sum_{r=1}^{s} u_r Y_{r0}}{\sum_{i=1}^{m} v_i X_{i0}} \qquad (6)$$

$$\begin{aligned} &\textit{subject to}: \\ &\frac{\sum_{r=1}^{s} u_r Y_{rj}}{\sum_{i=1}^{m} v_i X_{ij}} \leq 1 \quad j = 1, \ldots, n \\ &u_r, v_i \geq 0 \quad r = 1, \ldots, s; \; i = 1, \ldots, m. \end{aligned}$$

Therefore, in order to obtain the efficiency measure, the optimal value $z_0^*$, $g_0^*$, $f_0^*$, or $h_0^*$ should be found. Then only a linear programming problem is solved.

## 4. Application of DEA to the TTU.

evaluation considers enrollments only. Choos ing the data for the evaluation was simple but the data was not easily available in th TTU. However, the data was obtained wit some difficulty.

The crucial inputs of an academic uni consist academic staff of various ranks. Th total academic staff is expressed in teachin hours (TH) and consists of professorial sta (prof., associate prof.,and assistant prof.), in structor, and teaching assistant.

Teaching outputs can be measured in terr of enrollment in BSc, MSc, and PhD leve courses and can be expressed in student hour (SH).

Therefore, there are five inputs, and thre outputs.

The number of units actually used in th evaluation is 19. One of these is institute namely the Institute of Mathematics. The re maining 18 units are departments in the Fac ulties of Sciences, Literature and Humanities Education, and Physical Education.

The Institute of Educational Research an the Unit of Islamic Culture have different spe cific tasks, in that, they have not been con sidered by this evaluation.

DEA doesn't give meaningful results in a cases. If the number of inputs and outputs i large in comparison to the number of DMUs DEA will find an efficiency of 1 for many unit and so their efficiencies can't be compared ef

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*ber of inputs and outputs"* is as a rule which is stated by Charnes, Copper, and Rhodes (1978).

In our evaluation the number of units is 19, and the number of inputs and outputs is 8. However, when all eight inputs and outputs were included, no interesting results were obtained. Inputs and outputs were therefore combined in a number of ways. The members of professors of three ranks were added using different weights to give one input, and instructors and teaching assistant were also added using different weights to give another input. The proportions of salaries were considered as the weights. The graduate enrollments of two levels, M.Sc and Ph.D, were added with the weights of 1 and 2.

Using this combination the results were obtained in a model with two inputs, professorial staff and those holding the ranks of an instructor, and two outputs, course enrollments in undergraduate and graduate programs.

Table 2 gives the main results, the first column displays the educational efficiencies expressed in terms of theoretical hours plus practical hours of course work (THW) and the second column is expressed in terms of two times practical hours plus theoretical hours of course work (PHW). This table is sorted in terms of the THW column.

In the first column there are some units with an efficiency rating of 1, namely the Department of Physical Education, Foundations of Education, and the Institute of Mathematics. The units with the lowest efficiencies are Departments of Geology and Biology. Five units have the efficiencies of less than 60%.

In the second column, the practical hours of course work are considered as two times the theoretical hours, but the efficiency changes are not significant.

In Table 2 the top five academic units with their THWs are belong to the faculties of the Literature and Humanities, Physical Education, and Education. There is evidently a sharp variation in the efficiencies of the departments of a faculty. For example, in the faculty of Education the Department of Foundations of Education has a 100% efficiency, while the Department of Instructional Technology and Methods has only 58%. The average efficiencies of departmentalized faculties cover widely varying efficiencies of their departments.

Table 3 gives the efficiencies for one Institute and four Faculties. The average efficiencies for the faculties (with the consideration of their departments) have been calculated and sorted in the THW column.

When the practical hours of course work are considered as twice the theoretical hours of course work, the efficiencies of the Faculties of Physical Education, Education, and Sciences go up because of laboratories and practical hours of course works.

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| Department/Institute | THW | PHW |
|---|---|---|
| Persian Literature | 100% | 100% |
| Theology and Islamic Culture | 100% | 100% |
| Foreign Languages | 100% | 100% |
| Women Physical Education | 100% | 100% |
| Foundations of Education | 100% | 100% |
| Institute of Mathematics | 100% | 100% |
| Men Physical Education | 97% | 100% |
| Mathematics | 96% | 96% |
| History | 95% | 95% |
| Psychology | 89% | 92% |
| Guidance and Counseling | 85% | 93% |
| Social Sciences | 71% | 68% |
| Biology | 66% | 73% |
| Geography | 63% | 68% |
| Arabic Language and Literature | 58% | 59% |
| Instructional Technology and Methods | 58% | 58% |
| Physics | 54% | 61% |
| Geology | 45% | 57% |
| Chemistry | 45% | 55% |

Table 2

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| Faculty/Institute | THW | PHW |
|---|---|---|
| Institute of Mathematics | 100% | 100% |
| Faculty of Physical Education | 98% | 100% |
| Faculty of Literature and Humanities | 84% | 84% |
| Faculty of Education | 83% | 86% |
| Faculty of Sciences | 61% | 68% |

Table 3. Efficiency scores for 4 faculties and one institute.

This figure represents the efficiencies of the faculties in terms of THW and PHW. As the Figure indicates, Institute of Mathematics and Faculty of Physical Education have the highest efficiency rating and Faculty of Sciences has the lowest efficiency rating.

Evidently, the efficiencies of the faculties depends on the efficiencies of their departments. Thus, in this study some strategies are suggested for improving the efficiencies of the departments.

Having assured the accuracy of the accumulated data for evaluating the educational efficiencies of the academic units, the authors concentrated on analyzing the data.

The University has an academic staff of 297 to be exact. They are ranked in the professorial staff levels of (prof., associate prof., and assistant prof.), instructors, and teaching assistants, and are assigned in the academic units. The inefficiencies of the units are caused by the inefficient faculty members who ficiency and the officials in charge of this faculty state that the laboratory courses are the cause for their inefficiencies. Firstly, with due attention to this point that they assign the academic staff with the rank of teaching assistant in the laboratories and these academic staff have been given the lowest weight in the evaluation procedure. Secondly, it is not true that inefficiencies are confined to those departments with the laboratory courses (departments of Social Sciences, Arabic Language and Literature, Instructional Technology and Methods are inefficient as well). Therefore, laboratory courses are not justified for these departments' low efficiencies. However, the scores for the PHW are represented in table 2 and for the practical hours (laboratory, workshop, and so on) of course work are considered as two times the theoritical hours (i.e. equivalent to the work of the student hours); therefore, the evaluation remains unchanged and the same as before. Accordingly, the Fac-

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man power adjustment of the academic staff.

It is anticipated to retire some of the inefficient staff as the first part of this plan, in particular, those from the departments of Social Sciences, Biology, Geography, Arabic Language and Literature, Instructional Technology and Methods, Physics, Geology, and Chemistry (i.e. the departments with the efficiency scores of less than 80%).

As another part of the plan, it was suggested to circularize the academic departments in regard to the stipend of the academic staff. To authorize the departments' heads to assign the academic staff to a full-time, half-time, or part-time research activities. To upgrade the research as a full-time University activities. Furthermore, a circular is regulated for the maximum and the minimum number of students in a classroom in the general, basic, and specialized courses separately and the undergraduate and graduate levels (B. Sc., M. Sc., and Ph. D.) also separately.

It is useful to list the assumptions when DEA is applied in the evaluation of academic units. The following assumptions have been made:

1) The assumptions of linear programming hold.

2) Teaching of various courses have the same quality.

3) All courses require the same amount of re-

tional Departments, and the Computer Center of the TTU. It is clear that the validity of the results depend upon the correctness of the given data.

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## References

[1] Al-Faraj, T. N.; Alidi, A. S.; Bu-Bshait, K. A.; "Evaluation of Ban Branches by Means of Data Envelopment Analysis," International Journal of Operat & Production Management, 13(9), (1993), 45-52.

[2] Banker, R. D.; Charnes, A.; Cooper, W. W.,"Some Models for Estimating Technical and Scale Inefficiencies in Data Envelopment Analysis," Management Sci., 30, (1984), 1078-1092.

[3] Charnes, A.; Cooper, W. W.; Rhodes, E.,"Measuring the Efficiency of Decision Making Units," European J. Oper. Res., 2, (1978), 429-444.

[4] Valdmanis, V.; "Sensitivity Analysis for DEA Models: An Empirical Example Using Public vs. NFP Hospitals," Journal of Public Economics, 48(2), (1992),

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## Appendix A

| Department/Institute and **Faculty** | Input1 Inst. | Input2 Prof. | Output1 U. Grad. | Output2 Grad. |
|---|---|---|---|---|
| Persian Literature | 81.0 | 87.6 | 5191 | 205 |
| Theology and Islamic Culture | 85.0 | 12.8 | 3629 | 0 |
| History | 56.7 | 55.2 | 3302 | 0 |
| Geography | 91.0 | 78.8 | 3379 | 8 |
| Foreign Languages | 216.0 | 72.0 | 5368 | 639 |
| Arabic Language and Literature | 58.0 | 25.6 | 1674 | 0 |
| Social Sciences | 112.2 | 8.8 | 2350 | 0 |
| **Faculty of Literature** | **699.9** | **340.8** | **24893** | **852** |
| Men Physical Education | 293.2 | 52.0 | 6315 | 414 |
| Women Physical Education | 186.6 | 0.0 | 2865 | 0 |
| **Faculty of Physical Ed.** | **479.8** | **52.0** | **9180** | **414** |
| Mathematics | 143.4 | 105.2 | 7689 | 66 |
| Geology | 108.7 | 127.0 | 2165 | 266 |
| Biology | 105.7 | 134.4 | 3963 | 315 |
| Chemistry | 235.0 | 236.8 | 6643 | 236 |
| Physics | 146.3 | 124.0 | 4611 | 128 |
| **Faculty of Sciences** | **739.1** | **727.4** | **25071** | **1011** |
| Foundations of Education | 57.0 | 203.0 | 4869 | 540 |
| Instructional Technology | 118.7 | 48.2 | 3313 | 16 |
| Psychology | 58.0 | 47.4 | 1853 | 230 |
| Guidance and Counseling | 146.0 | 50.8 | 4578 | 217 |
| **Faculty of Education** | **379.7** | **349.4** | **14613** | **1003** |
| **Institute of Mathematics** | **0.0** | **91.3** | **0** | **508** |

Inputs and THW-Outputs for 19 Academic Units and 4 Faculties of TTU on the First Semester 1993-94.

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## Appendix B

| Department/Institute and **Faculty** | Input1 Inst. | Input2 Prof. | Output1 U. Grad. | Output2 Grad. |
|---|---|---|---|---|
| Persian Literature | 81.0 | 87.6 | 5191 | 205 |
| Theology and Islamic Culture | 85.0 | 12.8 | 3629 | 0 |
| History | 56.7 | 55.2 | 3302 | 0 |
| Geography | 91.0 | 78.8 | 3647 | 8 |
| Foreign Languages | 216.0 | 72.0 | 5509 | 647 |
| Arabic Language and Literature | 58.0 | 25.6 | 1674 | 0 |
| Social Sciences | 112.2 | 8.8 | 2635 | 0 |
| **Faculty of Literature** | **699.9** | **340.8** | **25587** | **852** |
| | | | | |
| Men Physical Education | 293.2 | 52.0 | 9382 | 454 |
| Women Physical Education | 186.6 | 0.0 | 4751 | 0 |
| **Faculty of Physical Ed.** | **479.8** | **52.0** | **14133** | **454** |
| | | | | |
| Mathematics | 143.4 | 105.2 | 7689 | 66 |
| Geology | 108.7 | 127.0 | 3086 | 312 |
| Biology | 105.7 | 134.4 | 4549 | 337 |
| Chemistry | 235.0 | 236.8 | 8079 | 245 |
| Physics | 146.3 | 124.0 | 5227 | 141 |
| **Faculty of Sciences** | **739.1** | **727.4** | **28630** | **1101** |
| | | | | |
| Foundations of Education | 57.0 | 203.0 | 4869 | 540 |
| Instructional Technology | 118.7 | 48.2 | 3313 | 16 |
| Psychology | 58.0 | 47.4 | 1853 | 260 |
| Guidance and Counseling | 146.0 | 50.8 | 5011 | 261 |
| **Faculty of Education** | **379.7** | **349.4** | **15046** | **1077** |
| | | | | |
| **Institute of Mathematics** | **0.0** | **91.3** | **0** | **532** |

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