On Weak McCoy Rings

E. Hashemi : Shahrood University of Technology

Abstract

In this note we introduce the notion of weak McCoy rings as a generalization of McCoy rings, and investigate their properties. Also we show that, if R is a semi-commutative ring, then R is weak McCoy if and only if $R[x]$ is weak McCoy.

1. Introduction

Throughout this paper, all rings are associative with identity. For a commutative ring R, McCoy [10] obtained the following result: If $f(x)g(x)=0$ for some non-zero polynomials $f(x), g(x) \in R[x]$, then $f(x)c=0$ for some non-zero $c \in R$. According to Nielsen [12], a ring R is called *right McCoy* whenever polynomials $f(x)$, $g(x) \in R[x]-\{0\}$ satisfy $f(x)g(x)=0$, there exists a non-zero $r \in R$ such that $f(x)r=0$. Left McCoy rings are defined similarly. If a ring is both left and right McCoy, we say that the ring is a *McCoy ring*. It is well known that commutative rings are always McCoy rings [10], but it is not true for non-commutative rings (see [12]).

Recall that a ring R is called:

reduced if $a^2=0 \Rightarrow a=0$, for all $a \in R$,

reversible if $ab=0 \Rightarrow ba=0$, for all $a,b \in R$,

symmetric if $abc=0 \Rightarrow acb=0$, for all $a,b,c \in R$,

semi-commutative if $ab=0 \Rightarrow aRb=0$, for all $a,b \in R$.

The following implications hold:

2000 Mathematics Subject Classification: 16N40, 16U80

Keywords: Semi-commutative ring, McCoy ring, Polynomial ring, Reversible ring, Matrix ring, Classical quotient ring

Received 5 Nov. 2008 Revised 19 Oct. 2009

eb_hashemi@yahoo.com

[Downloaded from isharat.khu.ac.ir on 2026-10-28]

reduced $\Rightarrow$ symmetric $\Rightarrow$ reversible $\Rightarrow$ semi-commutative.

Reversible rings are McCoy rings by [12]. But the converse is not true; there exists a non-reversible McCoy ring (see [12]).

Motivated by the above, as a generalization of McCoy rings, in this paper we introduce the notion of weak McCoy rings and investigate their properties and extend several known results relating to McCoy rings to a general setting.

For a ring R, we denote by $nil(R)$ the set of all nilpotent elements of R, by $N_*(R)$ the prime radical of R and by $M_n(R)$, $U_n(R)$ and $L_n(R)$ the $n\times n$ matrix ring over R, the $n\times n$ upper and lower triangular matrix rings over R respectively.

2. On Weak McCoy rings

Definition2.1. We say R is a *weak McCoy* ring if $f(x)g(x)\in nil(R[x])$ implies $f(x)c\in nil(R[x])$, for some non-zero $c\in R$, where $f(x)$ and $g(x)$ are non-zero polynomials in $R[x]$.

Remark 2.2. Since ab is nilpotent if and only if ba is nilpotent in a ring, hence the definition of weak McCoy rings is left-right symmetric.

Proposition 2.3. McCoy rings are weak McCoy.

Proof. Let R be a McCoy ring and $f(x)g(x)\in nil(R[x])$ for non-zero polynomials $f(x)$, $g(x)\in R[x]$. Then there exists $m,n\geq 1$, such that $(f(x)g(x))^n=(g(x)f(x))^m=0$, and $(f(x)g(x))^{n-1},(g(x)f(x))^{m-1}\neq 0$. If $f(x)g(x)=0$ or $g(x)f(x)=0$, then the result follows from the definition of McCoy rings. Assume $f(x)g(x)\neq 0\neq g(x)f(x)$ and $0=(f(x)g(x))^n=f(x)(g(x)f(x)\ldots f(x)g(x))=f(x)h(x)$.

If $h(x)=g(x)f(x)\ldots f(x)g(x)\neq 0$, then $f(x)c=0$ for some non-zero $c\in R$, since R is McCoy.

Let $h(x)=g(x)(f(x)g(x)\ldots f(x)g(x))=g(x)(f(x)g(x))^{n-1}=0.$ Since $(f(x)g(x))^{n-1}\neq 0$ and R is McCoy, there exists $0\neq d\in R$ such that $g(x)d=0$. Therefore $f(x)c=0$ or

[Downloaded from isharat.khu.ac.ir on 2026-10-28]

$g(x)d = 0$ for some non-zero $c, d \in R$. Hence $f(x)c \in nil(R[x])$ or $dg(x) \in nil(R[x])$ for some non-zero $c, d \in R$. Therefore R is weak McCoy.

Proposition 2.4. Let R be a ring. Then $U_n(R)$ and $L_n(R)$ are weak McCoy for each $n \geq 2$.

Proof. Clearly $U_n(R)[x] \cong U_n(R[x])$ and for each $A = \begin{bmatrix} 0 & f_{12} & \cdots & f_{1n} \\ 0 & 0 & \cdots & f_{2n} \\ \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & 0 & 0 \end{bmatrix} \in U_n(R[x])$, $A^n = 0$. Let $0 \neq A = \begin{bmatrix} f_{11} & f_{12} & \cdots & f_{1n} \\ 0 & f_{22} & \cdots & f_{2n} \\ 0 & 0 & \ddots & \cdots \\ 0 & 0 & \cdots & f_{nn} \end{bmatrix} \in U_n(R[x])$. Then

$$A\begin{bmatrix} 0 & 1 & \cdots & 1 \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \ddots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix} = \begin{bmatrix} 0 & g_{12} & \cdots & g_{1n} \\ 0 & 0 & \cdots & g_{2n} \\ \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} \text{ and } \left(A\begin{bmatrix} 0 & 1 & \cdots & 1 \\ 0 & 0 & \cdots & 1 \\ \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & 0 & 0 \end{bmatrix} \right)^n = 0.$$

Hence $U_n(R)$ is weak McCoy. By a similar argument one can show that $L_n(R)$ is weak McCoy.

Proposition 2.5. Let R and S be rings and ${}_RM_S$ a bimodule. Then $\begin{bmatrix} R & M \\ 0 & S \end{bmatrix}$ is a weak McCoy ring.

Proof. Similarly, as used in Proposition 2.4 one can prove it.

The following example shows that $U_n(R)$ and $M_n(R)$ are neither left nor right McCoy for some $n \geq 2$.

Example 2.6. Let R be a ring. We show that $U_4(R)$ and $M_4(R)$ are neither right nor left McCoy. Let $f(x) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} x$ and

[Downloaded from isharat.khu.ac.ir on 2026-10-28]

$$g(x)=\begin{bmatrix}0&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix}+\begin{bmatrix}0&1&0&0\\0&0&0&0\\0&0&0&1\\0&0&0&0\end{bmatrix}x\in U_4(R)[x]\subseteq M_4(R)[x]$$. Then $f(x)g(x)=0$.

If $f(x)A=0$, for some $A=\lfloor a_{ij}\rfloor\in M_4(R)$, then $0=\begin{bmatrix}1&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&0\end{bmatrix}A=\begin{bmatrix}a_{11}&a_{12}&a_{13}&a_{14}\\0&0&0&0\\a_{31}&a_{32}&a_{33}&a_{34}\\0&0&0&0\end{bmatrix}$

and $0=\begin{bmatrix}0&-1&0&0\\0&0&0&0\\0&0&0&-1\\0&0&0&0\end{bmatrix}A=\begin{bmatrix}-a_{21}&-a_{22}&-a_{23}&-a_{24}\\0&0&0&0\\-a_{41}&-a_{42}&-a_{43}&-a_{44}\\0&0&0&0\end{bmatrix}$. Hence $A=0$ and $U_4(R)$ and $M_4(R)$ are not right McCoy. If $Bg(x)=0$ for some $B\in M_4(R)$, then by a similar way as above, we can show $B=0$. Therefore $U_4(R)$ and $M_4(R)$ are not left McCoy.

Definition 2.7. A ring R is called *right Ore* if given $a,b\in R$ with b regular there exist $a_1,b_1\in R$ with b_1 regular such that $ab_1=ba_1$. It is well-known that R is a right Ore ring if and only if the classical right quotient ring of R exists. We use $C(R)$ to denote the set of all regular elements in R.

Theorem 2.8. Let R be a right Ore ring with its classical right quotient ring Q. If R is weak McCoy then Q is weak McCoy.

Proof. Let $0\neq F(x)=\sum_{i=0}^{m}a_iu^{-1}x^i$ and $0\neq G(x)=\sum_{j=0}^{n}b_jv^{-1}x^j$ with $a_i,b_j\in R, u,v\in C(R)$ such that $F(x)G(x)\in nil(Q[x])$.

Case1. $F(x)G(x)=0$ or $G(x)F(x)=0$. Assume that $F(x)G(x)=0$. Since R is right Ore, there exists $b_j^{'}\in R$ and $u_1\in C(R)$ such that $u^{-1}b_j=b_j^{'}u_1^{-1}$ for $j=1,\ldots,n$. Let $f(x)=\sum_{i=0}^{m}a_ix^i$ and $g(x)=\sum_{j=0}^{n}b_j^{'}x^j$ .Then $f(x)g(x)=0$. Since R is weak McCoy, there exists $0\neq c\in R$ with $f(x)c\in nil(R[x])\subseteq nil(Q[x])$. Hence $F(x)uc=f(x)u^{-1}uc=f(x)c\in nil(Q[x])$. If $G(x)F(x)=0$, then by a similar argument we can show that $G(x)vd\in nil(Q[x])$ for some non-zero $d\in R$.

[Downloaded from isharat.khu.ac.ir on 2026-10-28]

Case2. $F(x)G(x) \neq 0$ and $G(x)F(x) \neq 0$. Since $F(x)G(x) \in nil(Q[x])$, there exists $n \geq 2$ such that $(F(x)G(x))^n = 0$ and $(F(x)G(x))^{n-1} \neq 0$. Let $(F(x)G(x))^n = F(x)H(x)$. If $H(x) \neq 0$, then by a similar argument as above there exists $\alpha \in C(R)$, $r \in R$ such that $F(x)\alpha r \in nil(Q[x])$. Now assume $H(x) = G(x)\underbrace{F(x)G(x)\ldots F(x)G(x)}_{n-1} = 0$. Since $(F(x)G(x))^{n-1} \neq 0$ and R is weak McCoy, then by Case 1, there exists $\beta \in C(R)$, $s \in R$ such that $G(x)\beta s = 0$. Therefore Q is weak McCoy.

According to Bell [2], a ring R is called semi-commutative if $ab = 0$ implies $aRb = 0$. We say an ideal I is a *semi-commutative ideal*, if R/I is a semi-commutative ring.

Lemma 2.9. Let R be a semi-commutative ring. If $c_1c_2\cdots c_k = 0$ for some $c_i \in R$, then $c_1Rc_2Rc_3\cdots Rc_k = 0$.

Proof. By induction, let $c'_{k-1} = c_{k-1}c_k$. Then $c_1c_2\cdots c'_{k-1} = 0$ and by induction assumption, we have $0 = c_1Rc_2Rc_3\cdots Rc'_{k-1} = c_1Rc_2Rc_3\cdots Rc_{k-1}c_k$. Hence, for all $x \in c_1Rc_2Rc_3\cdots Rc_{k-1}$, we have $xc_k = 0$. It follows by hypothesis that $xRc_k = 0$. Thus $c_1Rc_2Rc_3\cdots Rc_k = 0$, as desired.

Lemma 2.10 (4, Lemma 2.5). Let R be a semi-commutative ring. Then $nil(R)$ is a semi-commutative ideal of R.

Proof. Let $a, b \in nil(R)$. Then $a^n = 0 = b^m$ for some $m, n \geq 0$. Each term of the expansion of $(a+b)^{m+n+1}$ has the form $x := (a^{i_1}b^{j_1})\cdots(a^{i_{m+n+1}}b^{j_{m+n+1}})$ where $i_r, j_s \in N \cup \{0\}$. Since $(i_1 + j_1) + (i_2 + j_2) + \ldots + (i_{m+n+1} + j_{m+n+1}) = \sum_{r=1}^{n} i_r + \sum_{s=1}^{m} j_s = m + n + 1$, either $\sum_{r=1}^{n} i_r \geq n$ or $\sum_{s=1}^{m} j_s \geq m$. If $\sum_{r=1}^{n} i_r \geq n$, then $a^{i_1}a^{i_2}\cdots a^{i_{m+n+1}} = 0$. Thus $(a^{i_1}b^{j_1})\cdots(a^{i_{m+n+1}}b^{j_{m+n+1}}) = 0$, by Lemma 2.9. If $\sum_{r=1}^{n} i_r < n$, then $\sum_{s=1}^{m} j_s \geq m$. Thus $b^{j_1}b^{j_2}\cdots b^{j_{m+n+1}} = 0$ and so $(a^{i_1}b^{j_1})\cdots(a^{i_{m+n+1}}b^{j_{m+n+1}}) = 0$, by Lemma 2.9. Hence $(a+b)^{m+n+1} = 0$.

Now suppose that $a^n = 0$ and $r \in R$. Then $(ar)^n = 0 = (ra)^n$, by Lemma 2.9. Thus $nil(R)$ is an ideal of R.

[Downloaded from isharat.khu.ac.ir on 2026-10-28]

Since $R/nil(R)$ is a reduced ring, hence it is a semi-commutative ring. Therefore $nil(R)$ is a semi-commutative ideal of R.

Lemma 2.11. Let R be a semi-commutative ring. Then $nil(R[x]) = nil(R)[x]$.

Proof. Let $f(x) = a_0 + ... + a_n x^n \in nil(R[x])$. Then $f(x)^k = 0$, for some integer $k \geq 0$. Hence $a_n{}^k = 0$, and that $a_n \in nil(R)$. There exists $g(x), h(x) \in R[x]$ such that $f(x)^k = (a_0 + ... + a_{n-1}x^{n-1})^k + a_n g(x) + h(x)a_n$. Since $nil(R)[x]$ is an ideal of $R[x]$ and $a_n g(x), h(x)a_n, f(x)^k \in nil(R)[x]$, we have $(a_0 + ... + a_{n-1}x^{n-1})^k \in nil(R)[x]$. Hence $a_{n-1}{}^k \in nil(R)$ and that $a_{n-1} \in nil(R)$. Continuing this process yields $a_0, ..., a_n \in nil(R)$. Therefore $nil(R[x]) \subseteq nil(R)[x]$.

Now, let $f(x) = a_0 + ... + a_n x^n \in nil(R)[x]$. Then $a_i^{m_i} = 0$, for some positive integer m_i. Let $k = m_0 + \cdots + m_n + 1$. Then $(f(x))^k = \sum (a_0^{i_{01}} (a_1 x)^{i_{11}} \cdots (a_n x^n)^{i_{n1}}) \cdots (a_0^{i_{0k}} (a_1 x)^{i_{1k}} \cdots (a_n x^n)^{i_{nk}})$, where $i_{0r} + \cdots + i_{nr} = 1$, for $r = 1, \cdots k$ and $0 \leq i_{rs} \leq 1$. Each coefficient of $f(x)^k$ is a sum of such elements $\gamma = (\ (a_0)^{i_{01}} \cdots\ (a_n)^{i_{n1}}) \cdots (\ (a_0)^{i_{0k}} \cdots\ (a_n)^{i_{nk}})$, where $i_{0r} + \cdots + i_{nr} = 1$.

It can be easily checked that there exists $a_k \in \{a_0, \cdots, a_n\}$ such that $i_{t1} + \cdots + i_{tk} \geq m_t$. Since $a_t^{m_t} = 0$ and R is semi-commutative, $\gamma = 0$. Thus $(f(x))^k = 0$ and $nil(R)[x] \subseteq nil(R[x])$. Therefore $nil(R[x]) = nil(R)[x]$.

Lemma 2.12 . Let R be a semi-commutative ring. Then $nil(R[x][y]) = nil(R[x])[y]$.

Proof. By Lemma 2.11, $nil(R[x])$ is an ideal of $R[x]$. Since $R[x]/nil(R[x])$ is a reduced ring, hence $nil(R[x])$ is a semi-commutative ideal of $R[x]$, and that $nil(R[x])[y] \subseteq nil(R[x][y])$.

Now, let $F(y) = \sum_{i=0}^{m} f_i y^i \in nil(R[x][y])$, where $f_i = \sum_{s=0}^{p_i} a_{is} x^s \in R[x]$. Then $F(y)^n = 0$, for some positive integers n. As in the proof of [1], let $k = n \sum \deg f_i$, where the degree is as polynomial in x and the degree of zero polynomial is taken to be 0. Then $(F(x^k))^n = 0$ and the set of coefficients of $F(x^k)$ is equal to the set of all coefficients of f_i, $0 \leq i \leq m$. Hence by Lemma 2.11, $a_{ij} \in nil(R)$ for all i, j and that $f_i \in nil(R[x])$, for each i. Thus $F(y) \in nil(R[x])[y]$. Therefore $nil(R[x][y]) = nil(R[x])[y]$.

[Downloaded from isharat.khu.ac.ir on 2026-10-28]

If R is semi-commutative, then $R[x]$ may not be semi-commutative, by [5, Example 2]). Here we will show that if R is semi-commutative, then R is weak McCoy if and only if $R[x]$ is weak McCoy.

Theorem 2.13. If R is a semi-commutative ring, then $R[x]$ is a weak McCoy ring if and only if R is weak McCoy.

Proof. Suppose that R is a weak McCoy ring. Let $F(t)=\sum_{i=0}^{m} f_i t^i$, $G(t)=\sum_{j=0}^{n} g_j t^j$ be non-zero polynomials in $R[x][t]$ such that $F(t)G(t)\in nil(R[x][t])$, where $f_i=\sum_{s=0}^{p_i} a_{is}x^s$, $g_j=\sum_{t=0}^{q_j} b_{jt}x^t \in R[x]$. As in the proof of [1], let $k=\sum \deg f_i+\sum \deg g_j$, where the degree is as polynomial in x and the degree of zero polynomial is taken to be 0. Then $F(x^k)=\sum_{i=0}^{m} f_i x^{ik}$, $G(x^k)=\sum_{j=0}^{n} g_j x^{jk}\in R[x]$, and the set of coefficients of the $F(x^k)$ is (respectively $G(x^k)$) equal to the set of all coefficients of f_i, $0\le i\le m$ (respectively g_j, $0\le j\le n$). Since $(F(t)G(t))^p=0$, for some $p\ge 1$, and x commutes with elements of R, $(F(x^k)G(x^k))^p=0$. Since R is weak McCoy, there is $0\ne r\in R$ such that $F(x^k)r\in nil(R[x])$ and $a_{is}r\in nil(R)$, $f_i r\in nil(R[x])$ for $0\le i\le m$, $0\le s\le p_i$ by Lemma 2.11. Hence $F(t)r\in nil(R[x][t])$, by Lemma 2.12. Therefore R[x] is weak McCoy.

Now suppose $R[x]$ is a weak McCoy ring and $f(t)g(t)\in nil(R[t])\subseteq nil(R[x][t])$. Since $R[x]$ is weak McCoy, there exists $0\ne h(x)\in R[x]$ such that $f(t)h(x)\in nil(R[x][t])$. Let $h(x)=a_0+\ldots+a_n x^n\in R[x]$ ($a_0\ne 0$). Then $f(t)a_0\in nil(R[t])$, since $(f(t)h(x))^k=(f(t)a_0)^k+k_1x+\ldots+k_{nk}x^{nk}$ with $k_1,\ldots,k_{nk}\in R[t]$. Therefore R is weak McCoy.

Theorem 2.14. Let R be a ring and Δ a multiplicatively closed subset of R consisting of central regular elements. Then R is weak McCoy if and only if $\Delta^{-1}R$ is weak McCoy.

[Downloaded from isharat.khu.ac.ir on 2026-10-28]

Proof. If R is is a weak McCoy ring, then by a similar way as used in Theorem 2.8, one can show that $\Delta^{-1}R$ is weak McCoy.

Conversely, let $\Delta^{-1}R$ be a weak McCoy ring. Let $f(x)=\sum_{i=0}^{m}a_ix^i$ and $g(x)=\sum_{j=0}^{n}b_j\ x^j$ be non-zero polynomials of $R[x]$ such that $f(x)g(x)\in nil(R[x])$. Since $\Delta^{-1}R$ is weak McCoy, $f(x)(c\alpha^{-1})\in nil((\Delta^{-1}R)[x])$ for some non-zero $c\alpha^{-1}\in\Delta^{-1}R$. Thus $f(x)c\in nil(R[x])$ and R is weak McCoy.

Corollary 2.15. Let R be a ring. Then $R[x]$ is weak McCoy if and only if $R[x,x^{-1}]$ is weak McCoy.

Proof. Clearly $\Delta=\{1,x,x^2,....\}$ is a multiplicatively closed subset of $R[x]$ consisting of central regular elements and $\Delta^{-1}R[x]=R[x,x^{-1}]$. Hence the proof follows from Theorem 2.14.

Theorem 2.16. The classes of weak McCoy rings are closed under direct limits.

Proof. Let $\mathrm{A}=\{R_i,\alpha_{ij}\}$ be a direct system of weak McCoy rings R_i for $i\in I$ and ring homomorphisms $\alpha_{ij}:R_i\to R_j$ for each $i\le j$ with $\alpha_{ij}(1)=1$, where I is a directed partially ordered set. Let $R=\varinjlim R_i$ be the direct limit of A with $\ell_i:R_i\to R$ and $\ell_j\alpha_{ij}=\ell_i$. We show that R is weak McCoy ring. Let $a,b\in R$. Then $a=\ell_i(a_i)$, $b=\ell_j(b_j)$ for some $i,j\in I$ and there is $k\in I$ such that $i\le k$, $j\le k$. Define $a+b=\ell_k(\alpha_{ik}(a_i)+\alpha_{jk}(b_j))$ and $ab=\ell_k(\alpha_{ik}(a_i)\alpha_{jk}(b_j))$, where $\alpha_{ik}(a_i)$, $\alpha_{jk}(b_j)\in R_k$. Then R forms a ring with $0=\ell_i(o)$ and $1=\ell_i(1)$. Let $f,g\in R[x]$ be non-zero polynomials such that $fg\in nil(R[x])$. There is $k\in I$ such that $f,g\in R_k[x]$. Hence $fg\in nil(R_k[x])$. Since R_k is weak McCoy, there exists $0\ne c_k\in R_k$ such that $fc_k\in nil(R_k[x])$. If $c=\ell_k(c_k)$, then $fc\in nil(R[x])$ with non-zero c. Therefore R is weak McCoy.

Proposition 2.17. (1) Let R be a ring. If there exists a non-zero ideal I of R such that $I[x]\subseteq nil(R[x])$, then R is weak McCoy.

[Downloaded from isharat.khu.ac.ir on 2026-10-28]

(2) Every non-semiprime ring is weak McCoy.

(3) Let R be a ring with a non-zero nilpotent ideal. Then $Mat_n(R)$ $(n \geq 2)$ is weak McCoy.

Proof. (1) Let $0 \neq f \in R[x]$. If $f \in I[x]$, then $fr \in nil(R[x])$ for all $r \in R$. If $f \notin I[x]$ then $fs \in I[x] \subseteq nil(R[x])$ for all non-zero $s \in I$. Thus R is weak McCoy.

(2) Let R be a ring with $N_*(R) \neq 0$. Since $0 \neq N_*(R)[x] = N_*(R[x]) \subseteq nil(R[x])$, R is weak McCoy by (1).

(3) Since $Mat_n(R)$ is non-semiprime, hence by (1) $Mat_n(R)$ is weak McCoy.

Acknowledgement

The author thanks the anonymous referees for their valuable suggestions, which simplified the proof of Lemma 2.10. This research is supported by the Shahrood University of Technology.

References

1. D.D. Anderson, V. Camillo, *Armendariz rings and Gaussian rings*, Comm. Algebra, 26(7)(1998) 2265-2272.
2. H.E. Bell, *Near-rings in which each element is a power of itself*, Bull. Austral. Math. Soc. 2 (1970) 363–368.
3. V. Camillo, P.P. Nielsen, *McCoy rings and zero-divisors*, J. Pure Appl. Algebra 212 (2008) 599-615.
4. E. Hashemi, *On ideals which have the weakly insertion of factor property*, Journal of Sciences, Islamic Republic of Iran 19(2) (2008) 145-152.
5. C. Huh, Y. Lee and A. *Smoktunowicz, Armendariz rings and semi-commutative rings*, Comm. Algebra. 30(2) (2002) 751-761.
6. O.A.S. Karamzadeh, *On constant products of polynomials*, Int. J. Math. Edu. Sci. Technol., 18(4) (1987) 627-629.
7. J. Krempa, *Some examples of reduced rings*, Algebra Colloq. 3 (1996) 289-300.

[Downloaded from isharat.khu.ac.ir on 2026-10-28]

8. T.Y. Lam, A . Leory, J. Matczuk, Primeness, *semiprimeness and the prime radical of Ore extensions*, Comm. Algebra, 25(8) (1997) 2459-2516.

9. Z. Liu and R.Y. Zhao, *On weak Armendariz rings*, Comm. Algebra, 34 (2006) 2607- 2616.

10. N.H. McCoy, *Remarks on divisors of zero*, Amer. Math. Monthly, 49 (1942) 286-295.

11. J.C. McConnell and J.C. Robson, *Non-commutative Noetherian Rings*, John Wiley & Sons Ltd., 1987.

12. P.P. Nielsen, *Semi-commutativity and the McCoy condition*, J. Algebra, 298 (2006)134-141.

13. L. Weiner, *Concerning a theorem of McCoy*, Amer. Math. Monthly 59(5) (1952)1281-1294.

[Downloaded from isharat.khu.ac.ir on 2026-10-28]